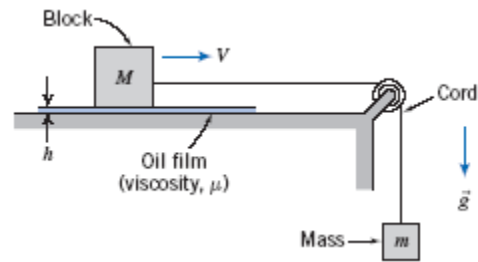


Problem 2.50

[Difficulty: 3]

2.50 A block of mass M slides on a thin film of oil. The film thickness is h and the area of the block is A . When released, mass m exerts tension on the cord, causing the block to accelerate. Neglect friction in the pulley and air resistance. Develop an algebraic expression for the viscous force that acts on the block when it moves at speed V . Derive a differential equation for the block speed as a function of time. Obtain an expression for the block speed as a function of time. The mass $M = 5 \text{ kg}$, $m = 1 \text{ kg}$, $A = 25 \text{ cm}^2$, and $h = 0.5 \text{ mm}$. If it takes 1 s for the speed to reach 1 m/s , find the oil viscosity μ . Plot the curve for $V(t)$.

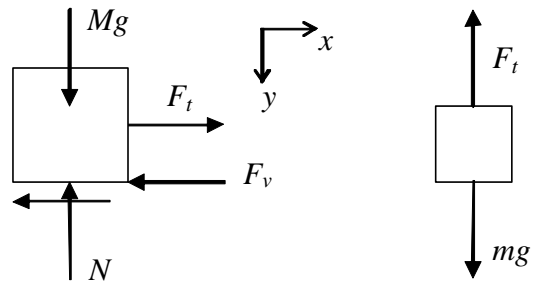


Given: Block on oil layer pulled by hanging weight

Find: Expression for viscous force at speed V ; differential equation for motion; block speed as function of time; oil viscosity

Solution:

Governing equations: $\tau_{yx} = \mu \frac{du}{dy}$ $\Sigma F_x = M \cdot a_x$



Assumptions: Laminar flow; linear velocity profile in oil layer

The given data is $M = 5 \cdot \text{kg}$ $W = m \cdot g = 9.81 \cdot \text{N}$ $A = 25 \cdot \text{cm}^2$ $h = 0.05 \cdot \text{mm}$

Equation of motion (block) $\Sigma F_x = M \cdot a_x$ so $F_t - F_v = M \cdot \frac{dV}{dt}$ (1)

Equation of motion (block) $\Sigma F_y = m \cdot a_y$ so $m \cdot g - F_t = m \cdot \frac{dV}{dt}$ (2)

Adding Eqs. (1) and (2) $m \cdot g - F_v = (M + m) \cdot \frac{dV}{dt}$

The friction force is $F_v = \tau_{yx} \cdot A = \mu \cdot \frac{du}{dy} \cdot A = \mu \cdot \frac{V}{h} \cdot A$

Hence $m \cdot g - \frac{\mu \cdot A}{h} \cdot V = (M + m) \cdot \frac{dV}{dt}$

To solve separate variables $dt = \frac{M + m}{m \cdot g - \frac{\mu \cdot A}{h} \cdot V} \cdot dV$

$$t = -\frac{(M + m) \cdot h}{\mu \cdot A} \cdot \left(\ln \left(m \cdot g - \frac{\mu \cdot A}{h} \cdot V \right) - \ln(m \cdot g) \right) = -\frac{(M + m) \cdot h}{\mu \cdot A} \cdot \ln \left(1 - \frac{\mu \cdot A}{m \cdot g \cdot h} \cdot V \right)$$

Hence taking antilogarithms $1 - \frac{\mu \cdot A}{m \cdot g \cdot h} \cdot V = e^{-\frac{\mu \cdot A}{(M + m) \cdot h} \cdot t}$

Finally

$$V = \frac{m \cdot g \cdot h}{\mu \cdot A} \cdot \left[1 - e^{-\frac{\mu \cdot A}{(M+m) \cdot h} \cdot t} \right]$$

The maximum velocity is $V = \frac{m \cdot g \cdot h}{\mu \cdot A}$

In Excel:

The data is

M = 5.00 kg
m = 1.00 kg
g = 9.81 m/s²
0 = 1.30 N.s/m²
A = 25 cm²
h = 0.5 mm

To find the viscosity for which the speed is 1 m/s after 1 s
use *Goal Seek* with the velocity targeted to be 1 m/s by varying
the viscosity in the set of cell below:

t (s)	V (m/s)
1.00	1.000

t (s)	V (m/s)
0.00	0.000
0.10	0.155
0.20	0.294
0.30	0.419
0.40	0.531
0.50	0.632
0.60	0.722
0.70	0.803
0.80	0.876
0.90	0.941
1.00	1.00
1.10	1.05
1.20	1.10
1.30	1.14
1.40	1.18
1.50	1.21
1.60	1.25
1.70	1.27
1.80	1.30
1.90	1.32
2.00	1.34
2.10	1.36
2.20	1.37
2.30	1.39
2.40	1.40
2.50	1.41
2.60	1.42
2.70	1.43
2.80	1.44
2.90	1.45
3.00	1.46

